

1 Introduction

 Name

Learning objectives

- Define and identify convex sets
- Define and identify convex functions
- Define the main elements of an optimization problem
- Formulate a generic optimization problem
- Differentiate between local and global optima
- Identify optimization problems in which the global optima can be computed
- Formulate simple optimization problems
- Solve simple optimization problems using GAMS
- Enumerate the special features of electricity as a commodity

Bibliography

- Basic reading:
 - Boyd, Stephen P., and Lieven Vandenberghe. Convex optimization. Cambridge university press, 2004 (chapters 2, 3, 4).
 - Gomez-Exposito, Antonio, Antonio J. Conejo, and Claudio Cañizares, eds. Electric energy systems: analysis and operation. CRC Press, 2008 (chapters 1 y 5).
 - A GAMS tutorial
 - GAMS - A user guide

 Draw a convex and non-convex set in $\mathbb{R}^2$

 Definition of a convex set

 Definition of norm

Example 1-1: Is a norm ball a convex set?

Using the properties of a norm function, determine whether a norm ball of radius r and center x_c defined as the set $\{x \mid \|x - x_c\| \leq r\}$ is convex.

List some typical convex sets

How can you check whether a set is convex using a computer?

 Draw a convex and non-convex function in $\mathbb{R}^2$

 Formal definition of a convex function

☰ List some typical convex functions

🖥️ How can you check whether a function is convex using a computer?

 Formulate a generic optimization problem identifying its main elements

 List some typical optimization problems

 Formal definitions

- Feasible point:
- Optimal value:
- Optimal point:
- Locally optimal:

Definition of convex optimization problem

Local and global optima

List and explain the most common types of convex optimization problems

Example 1-2: Generation scheduling

We consider a generating unit with whose production cost can be expressed as $\mathcal{C}(x) = ax^2 + bx + c$, where x represents the production level at MWh, and a , b and c are known parameters. The generating unit has a maximum and minimum output levels of $\underline{x}$ and $\bar{x}$, respectively. If the electricity price is denoted by p , formulate an optimization problem that determines the optimal generation level to maximize the profit made by the generating unit.

Solving Example 1-2 with GAMS (Example1-2.gms)

Formulate Example 1-2 in GAMS and solve it for different price values using the following data: $\underline{x} = 0$, $\bar{x} = 100$, $a = 0.2$, $b = 10$, $c = 100$. Fill in the following table with the obtained results and comment on them.

p (€/MWh)	10	20	30	40	50
x (MWh)					

📌 Example 1-3: Economic dispatch

We consider an economic dispatch problem (ED) in which the production schedules ($x_g, g = 1, \dots, N^G$) of different generating units are determined to satisfy a given demand (d) at the minimum cost. As in Example 2, the output of each unit is constrained by $\underline{x}_g$ and $\bar{x}_g$, and its operation cost is computed by the function $\mathcal{C}_g(x_g) = a_g x_g^2 + b_g x_g + c_g$. Formulate an optimization problem corresponding to an economic dispatch. Which kind of problem is this? Can we always find the global optimal point?

📄 Solving Example 1-3 with GAMS (Example1-3.gms)

Formulate Example 1-3 in GAMS 1 and solve it for different demand values using the following data:

Unit	$\underline{x}_g$	$\bar{x}_g$	a_g	b_g	c_g
g_1	0	250	0.01	5	100
g_2	0	150	0.02	10	100
g_3	0	100	0.03	16	100

Fill in the following table with the obtained results and comment on them.

d (MWh)	100	200	300	400	500
x_{g1} (MWh)					
x_{g2} (MWh)					
x_{g3} (MWh)					

📌 Example 1-4: Economic dispatch (with network)

In Example 1-3 all generating units and loads are assumed to be located at the same node. However, electricity and other commodities are generally produced far from the consumption center and therefore, a network is required for its transportation. In this example, apart from the generating units and the demand, we also consider a number of nodes or buses ($n = 1, \dots, N^B$) and transmission lines ($l = 1, \dots, N^L$). The parameter h_{gn} is a unit-bus indicator equal to 1 if unit g is located at bus n , and 0 otherwise. The demand at each node is d_n , the flow through line l is denoted by f_l (with a maximum value of $\bar{f}_l$), and i_{ln} is an line-bus indicator parameter equal to 1/-1 for the sending/receiving bus of line l and to 0 otherwise. Formulate the economic dispatch considering network.

📄 Solving Example 1-4 with GAMS (Example1-4.gms)

Formulate Example 1-4 in GAMS and solve it using the following data of a 3-bus system:

Unit	Bus	$\underline{x}_g$	$\bar{x}_g$	a_g	b_g	c_g	Bus	d_n	Line	$\bar{f}_l$
g_1	n_1	0	250	0.01	5	100	n_1	0	$l_1(n_1 - n_2)$	50
g_2	n_2	0	250	0.02	10	100	n_2	0	$l_2(n_1 - n_3)$	100
g_3	n_3	0	250	0.03	16	100	n_3	250	$l_3(n_2 - n_3)$	100

Fill in the following table with the obtained results and comment on them.

Unit	x_g	Line	f_l	Cost
g_1		l_1		
g_2		l_2		
g_3		l_3		



Discuss whether the model in Example 1-4 could be used for other commodities. What differentiates electricity from other commodities?



Example 1-5: Economic dispatch (with DC-network)

Modify the formulation of Example 1-4 knowing that in electricity network, the flow between two nodes is defined as:

$$f_l = B_l \sum_n i_{ln} \delta_n$$

where B_l is the line susceptance, and δ_n is the voltage angle at bus n .

Solving Example 1-5 with GAMS (Example1-5.gms)

Formulate Example 1-5 in GAMS and solve it using the following data:

Unit	Bus	$\underline{x}_g$	$\bar{x}_g$	a_g	b_g	c_g	Bus	d_n	Line	B_l	$\bar{f}_l$
g_1	n_1	0	250	0.01	5	100	n_1	0	$l_1(n_1 - n_2)$	100	50
g_2	n_2	0	250	0.02	10	100	n_2	0	$l_2(n_1 - n_3)$	100	100
g_3	n_3	0	250	0.03	16	100	n_3	250	$l_3(n_2 - n_3)$	100	100

Fill in the following table with the obtained results and compare them with those corresponding to Example 1-4. Discuss why the results are different.

Unit	x_g	Line	f_l	Cost
$g1$		l_1		
$g2$		l_2		
$g3$		l_3		